INTERNOISE

Budapest

August 1997
Abstract

From the early 1970’s Dassault Aviation has developed its own Finite Element code and CAD system. This tool has freely evolved, ‘daily shaped’ by their users. This paper presents how our methods today match our needs and try to analyse the leading characteristics of the proposed solving methods. A full dynamic model of a plane including structural dynamics, interior acoustics, exterior acoustics in moving media, servo-command system and identification are considered.

At first, the complexity of our problem is discussed. We underline that fluid/structure coupling is critical in aeronautics for it leads directly to wing dimensioning. We also note that external load configurations of military aircraft involve numerous mass distribution cases and that the whole flight domain should be explored.

Then, our computational strategy is presented. The main idea is to first built independent (uncoupled) reduced dynamic subsystems to allow, afterwards, a systemic approach of the coupled problem. The snag of this approach is the need of advanced methods in dynamic substructuring and in coupling techniques. Only original points give rise to theoretical development. In particular, a solution of linear acoustics in moving media is given. Modal projection, load basis, monomial basis and incompatible coupling operators are explored.

Untill today, we find these methods to be the only way to resolve our problems taking into account computer capabilities. With the increase in performance of computer these ‘sparse’ methods remain very attractive for we expect them to be used not only for simulation but for design purposes.
1. INTRODUCTION

In an industrial context, dynamic substructuring and modal synthesis is an attractive way to study complex dynamic systems. It introduces a new step in the modelisation process which allows, in the field of aeroelasticity and vibroacoustics, the study of an aircraft within various dynamic configurations. The snag to this approach is the need of advanced methods in dynamic substructuring and in coupling techniques. Moreover, users of such methods are compelled to have know-how in this domain. This paper addresses theoretical aspects of methods for dynamics daily used in our CAD/CAM system Catia-Elfini [1]. In particular, a set of reduced basis built around
natural vibration modes, static loads and static displacements are explored for bounded domains (structure and interior acoustics) while monomial basis are introduced for exterior problems (external acoustics). A procedure to take into account rigid motions of the plane is also explained.

2 STRUCTURAL REDUCED BASIS

Let $X$ be the Finite Element nodal displacement vector of a substructure (S), the dynamic equation of this structure is written as $M\ddot{X} + KX = F$ (1), with $M$ the mass matrix, $K$ the stiffness matrix and $F$ external loads. Through a chosen base transform of matrix $V$, $X$ is expressed as $X = VX_i$ (2), where $x$ becomes a new state vector of a smaller size. Considering a test function $X^*$ discretized as $X$ and pre-multiplying equation (1) by $'X^*$ leads for any $X^*$ to the reduced dynamic subsystem

$$mx + kx = f$$

with $m = VMV^t$, $k = VMV^t$, $f = VF$. The aim is to form equation (3) for each subsystem (see fig. 1) before coupling them to deal with the entire system. Moreover, for an aeroplane, it is convenient to express $F = VF^tX$ as

$$X^* = F$$

which account for rigid motions, normal modes (elastic modes), frontier modes and partial-rigid motions of control surfaces as aileron.

2.1 Modal Basis and Frontier Degrees of Freedom

Now the problem arises as how to built the $V$ matrix of equation (2). To exhibit this matrix for the different kinds of reduced basis we have developed, let us degenerate our problem eliminating rigid motions and considering an external boundary (B) where the structure is clamped while (F) denotes the internal frontier. Thus $X = V_i x_i + V_f x_f$ (5).

Doing a partition of the Finite Element unknown $X$ between internal degrees $X_i$ and frontier degrees $X_f$, equation (5) is rewritten,

$$\begin{bmatrix} X_i \\ X_f \end{bmatrix} = \begin{bmatrix} V_{ie} & V_{if} \\ V_{fe} & V_{ff} \end{bmatrix} \begin{bmatrix} x_i \\ x_f \end{bmatrix}. \quad (6)$$

Three kinds of models are available in Catia-Elfini software, namely M1, M2 and M3. M1 and M2 are basic models often referred respectively as fixed-constraint mode and free-constraint mode [2] while model M3 is a in house model equivalent to model M1 but built through model M2.

The model M1 is constructed from static deflexion shapes at the frontier nodes. It means that $x_f = X_f$ in equation (6). Thus identifying terms of (6), $V_{ie}$ is null and $V_{ff}$ is equal to the identity matrix. Now letting all the $x_f = X_f$ be null, the fist row of system (6) gives $X_i = V_{ie} x_i$ which proves that $V_{ie}$ has to be the matrix of the normal modes when the frontier is clamped. The only remaining term $V_{ff}$ can be deduced considering the static solution of (1), $KX = F$, while imposing successively an unitary displacement at each $X_f$. Recalling relation (6) with updated values, we obtain,

$$\begin{bmatrix} K_{ii} & K_{if} \\ K_{fi} & K_{ff} \end{bmatrix} \begin{bmatrix} V_{if} \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (7)$$

which yields to $V_{ff} = -K_{ii}^{-1}K_{if}$. 

The model M2 is built from static loads at each frontier nodes. In this case \( x_f = F_f \) in equation (6). If \( x_f = F_f \) is set to a null value (i.e. \( X_f \) free) then the first row of equation (6) reduces to \( X = V_x x_e \), which proves that the normal modes with a free frontier have to be used for this model. We notice that the second row of (6) gives a non null value for \( V_{fe} \) whereas this term was null in model M1. \( V_{if} \) and \( V_{ff} \) are determined using static solutions when imposing successively unitary force loads at the frontier. Updating equation (6) with the previous found values, \( K K = F \) becomes,

\[
\begin{bmatrix}
K_{ii} & K_{if} \\
K_{fi} & K_{ff}
\end{bmatrix}
\begin{bmatrix}
V_{if} \\
V_{ff}
\end{bmatrix}
= \begin{bmatrix}
0 \\
I
\end{bmatrix},
\]

which leads to \( V_{if} = -K_{ii}^{-1} K_{if} \) and \( V_{ff} = K_{ff}^{-1} \) with \( K_{ff} = -K_{ff} K_{ii}^{-1} K_{ff} + K_{ff} \).

The model M3 is the most popular at Dassault Aviation. Let \( V_1 \) and \( V_2 \) be the transform matrix of the previous models M1 and M2, and \( V_3 \) the transform matrix we intend to determine for this model. M3 is deduced from model M2 applying the square transform matrix \( T \) so that the reduced co-ordinates of M3 and M2 are linked by \( X = V_3 x_3 = V_2 T x_1 \).

Next, static displacement frontier modes are imposed to M3 as in model M1. Thus, as we have seen for model M1, the internal degrees of M3 are necessarily fixed-constraint component modes, which can be translated as \( x_2 = x_3 \). Hence, \( V_2 T = V_1 \) (9) will give the \( T \) matrix. Using results of model M1 and M3 and identifying obvious terms we find \( T \) obeying to equation (9) as,

\[
\begin{bmatrix}
V_{2,ie} \\
V_{2,fe}
\end{bmatrix}
\begin{bmatrix}
-K_{ii}^{-1} K_{if} K_{ff}^{-1} \\
-K_{ff}^{-1}
\end{bmatrix}
\begin{bmatrix}
I \\
0
\end{bmatrix}
= \begin{bmatrix}
V_{1,ie} \\
0
\end{bmatrix},
\]

with \( T_{fe} = -K_{ff} V_{2,fe} \) (11).

2.2 Rigid Motions

In this section, the external Boundary (B) no longer exists and the question is how to take into account rigid motions when using model M1, M2 or M3. In fact, no special procedure is needed for model M1 as long as frontier degrees of freedom (dof) do decompose rigid motions which (with obvious notations where \( m_r \) is the mass matrix of rigid motions) can be found as \( x_r = m_r^{-1} V_r' M V_r x_1 \). The problem differs for model M2 and M3 for both models are built considering (F) as free, \( x_r \) becomes an explicit degree of the basis ( \( X = V_x x_r + V_x x_e \) (12)), and the substructure is first calculated with isostatic conditions. The elastic motion \( X_e \) is written as \( X_e = X_{iso} + V_r' \bar{\xi}_r \) (13), where the vector \( \bar{\xi}_r \) is computed for \( \bar{\xi}_r \) to be orthogonal to rigid motions, \( V_r' M X_r = 0 \) (14). Substituting (13) into (14) gives \( X_r = [I - M V_r'^{-1} m_r V_r'] X_{iso} = S X_{iso} \) (where \( S \) is called the matrix of solid motion correction) which follows \( X = V_x x_r + V_r' A V_{iso} x_{iso} \) with \( V_r' A V_{iso} \) identified recalling (12) to \( V_r \) and \( x_{iso} \) to \( x_r \). Next, if frontier modes are taken into account in M2 the components of \( V_f \) are orthogonalized to rigid modes \( (V_f' M V_f = 0) \) and the new term in \( T \) are obvious or similar to (11) with a split between \( x_r \) and \( x_e \),

\[
T_{fr} = -K_{ff} V_{2,fr} \quad \text{and} \quad T_{fe} = -K_{ff} V_{2,fe}.
\]

Partial Rigid motions (see equation 4) are also introduced to modelise control surfaces actions. They have no particular proprieties in regard to the reduced basis but a great interest when computing aerodynamic loads and plane equilibrium.
3 ACOUSTICAL REDUCED BASIS

3.1 Modal Basis
For bounded domains as the passenger cabin (see fig. 2), where stationary waves appears, the Helmholtz equation is discretized with Acoustic Finite Elements and the pressure $P$ is expressed directly through normal modes with a rigid motion as $P = V_p p$. With analogy to structural equation (3), the resulting reduced system is $m_p \ddot{p} + k_p p = \rho \ddot{u}$ (with $\rho$, the fluid density and $u=V_p U$, the reduced acoustic displacements).

![figure 2. Elasto-Acoustic Modal Synthesis - Modal Pressure Field Inside a Cabin](image)

3.2 Monomial Basis
For the unbounded domain representing the exterior of the plane, a Boundary Element Method is used taking into account the planes speed as a uniform fluid motion. Symbolically the pressure $P$ on the boundary is given for a displacement $U$ of the boundary by $P = \Phi [A] U$ with $A$ an aerodynamic transfer function. To form a reduced system, linearity (acoustic hypothesis) is considered and $P$ is pre-computed for a set of $n$ basic displacements $Q$ of monomial shape as $[P_1,...,P_n] = \Phi [A][Q_1,...,Q_n]$. Thus any further displacement $U$ expressed as $U = [Q_1,...,Q_n] q$ leads without calculations to $P = [P_1,...,P_n] q$.

4 CONCLUSION

Finally, the only remaining task is to merge the reduced uncoupled models previously presented to study various configurations of an aeroplane. This will be done writing the conservative of forces and displacements at the interfaces. Coupling between structural models or external systems (which generally act locally) will be supported through frontier dof whereas internal degrees will be used for surfacic interactions. In the same way, frontier dof will contribute to basis enrichment when computing responses, rigid motions dof will allow to take into account flight dynamics and monomial basis will ensure the independence of aerodynamics when studying several mass distributions on the plane.